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DECAY OF ELECTRIFIED ROTATING CAPILLARY JETS

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A study was made of the effect of electrification and rotation of the nonmonodisperse decay of capillary jets. The study was conducted within the framework of the complete system of equations of hydrodynamics by the Bubnov–Galerkin method.

The organization of a monodisperse spray is extremely important in a number of processes which take place in the power-generation and construction industries and in various types of sprayers commonly used in agriculture and other sectors. Here, an important role is played by the electrification of capillary jets [1-10]. Well-known theoretical studies have focused mainly on linear [1-6], weakly nonlinear [7], or finite-amplitude [10] stability in quiescent electrified jets.

In the present investigation, we examine the effect of rotation on the nonmonodisperse atomization of electrified capillary jets.

We will assume that the liquid is inviscid, incompressible, and ideally conducting. We will further assume that the flow inside the jet is a potential flow. The velocity potential of the main flow and the potential of a charged circular cylinder φ_0 have the form

$$\Phi_0 = \Gamma\theta/(2\pi),\ \varphi_0 = A\ln(r/a).$$

The equations, the boundary conditions, and the initial conditions appear as follows in dimensionless variables [10, 11]:

$$\Delta\Phi = 0\,(0 \leqslant r \leqslant r_* \equiv 1 + \xi)\ \Delta\varphi = 0\,(r \geqslant r_*); \tag{1}$$

$$\xi_t = \Phi_r - \xi_z\Phi_z,\ \varphi = 0\,(r = r_*,\ r \to \infty); \tag{2}$$

$$\Phi_t = -\frac{1}{2}\left[\Phi_r^2 + \Phi_z^2 + \omega\left(\frac{1}{r^2} - 1\right)\right] - (\varkappa - 1) + b(t)[\varphi_n^2 - 1](r - r_*); \tag{3}$$

$$\xi(z,\ 0) = \xi_0(z),\ \Phi(r,\ z,\ 0) = \Phi_0(r,\ z). \tag{4}$$

Here, $\varkappa \equiv [1 + \xi_t^2]^{-3/2}\left\{\left[\frac{1 + \xi_z^2}{1 + \xi}\right] - \xi_{zz}\right\}$, $b(t) = \left(\frac{a^2}{2\pi Ta}\right) = q^2\pi$, $\omega = \frac{\Gamma^2\rho^1}{(2\pi^2 Ta)}$; φ is the perturbation of electric potential; Q(t) is the surface charge per unit length of the jet at the given moment of time.

Conditions reflecting the boundedness of the perturbations of all of the physical quantities must also be satisfied.

We will use the method described in [10] to solve the stated problem. The sought solution is represented in the form

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$$\Phi = \sum_{n=0}^{N} I_0(n\alpha r)[\Phi_n^{(1)}(t)\cos n\alpha z + \Phi_n^{(2)}(t)\sin n\alpha z], \quad (5)$$

$$\varphi = A\sum_{n=0}^{N} K_0(n\alpha r)[\varphi_n^{(1)}(t)\cos n\alpha z + \varphi_n^{(2)}(t)\sin n\alpha z], \quad (6)$$

$$\xi = \sum_{n=0}^{N} [\xi_n^{(1)}\cos n\alpha z + \xi_n^{(2)}(t)\sin n\alpha z]. \quad (7)$$

Here, $I_0(x)$ and $K_0(x)$ are modified Bessel functions of the first and second kinds, respectively. Equations to determine $\Phi_n^{(i)}$ $\varphi_n^{(i)}\xi_n^{(i)}$ are obtained by inserting (5-7) into boundary conditions (2-3) and performing the standard orthogonalization procedure.

The value of A is determined from the relation [10]

$$(1/2)\int_0^{2\pi} (\varphi + A\ln r)_{nr_*}^2 f dz = (2\pi/\alpha) q, \quad f \equiv r_* \sqrt{1+\xi_z^2}.$$

Before proceeding to a discussion of the numerical results, let us examine the original problem in a linear formulation:

$$\xi_t = \Phi_r, \ \tilde{\varphi} = -\xi\,(r=1), \quad (8)$$

$$\Phi_t = \omega\xi + \xi + \xi_{zz} + (\tilde{\varphi}_n - \xi)\,2q^2\pi\,(r=1), \quad (9)$$

$$\Delta\Phi = 0\,(0 \leqslant r \leqslant 1), \ \Delta\varphi = 0\,(1 \leqslant r). \quad (10)$$

The sought solution has the following form: $\xi = \xi_1 \exp\sigma t \cos\alpha z$, $\Phi = \Phi_1 \exp\sigma t I(\alpha r)$ $\cos\alpha z$ $\varphi = \varphi_1 \exp\sigma t K_0(\alpha r)\cos\alpha z$. Inserting this solution into (8-10), we readily obtain: $\sigma^2(q, \omega) = \alpha I_0'(\alpha)\{1 - \alpha^2 + \omega - 2q^2[1 + \alpha K_0'(\alpha)]K_0(\alpha)\}I_0(\alpha)$.

It was shown in [10, 12] that both swirling [12] and the presence of a charge [10] on the surface of the jet lead to an increase in the value of the growth factor σ and a decrease in the size of the resulting droplets. There is also an expansion of the range of unstable wave numbers in this case.

As was shown by an analysis of $\sigma(q, \omega)$, with the simultaneous action of a charge and swirling, and "additive" effect is achieved and the values of the coefficients increase accordingly. For example, with $q = 1$ and $\omega = 1.44$, the coefficient σ roughly triples compared to the case $\omega = 0$. The characteristic wave numbers also increase by a factor of approximately 1.5 in this instance.

We first performed a series of procedural calculations for two types of initial data: $\xi(z, 0) \equiv \xi_1 = \delta_c \cos\alpha z$ and $\xi_2 \equiv \xi(0 \le z \le z_1, 0) = \delta z/z_1$, $\xi(z_1 \le z \le \pi, 0) = \delta - (z - z_1)\delta/(\pi - z_1)$; $\xi(z > \pi, 0) = \xi(2\pi - z)$; $\Phi(r, z, 0) = 0$; $\varphi(r, z, 0) = 0$. These calculations established that N = 6 basis functions are sufficient to reliably reproduce the forms of the surface of the jet (to with about 1%) in the parameter ranges $0 \le \omega \le 2$ and $0 \le q \le 1.4$. All of the results cited below were thus obtained for N = 6.

We will initially examine the effect of the wave number α on the mode of decay of the jet at q = 0 with an initial perturbation conforming to a cosine law (curves 1-4, Fig. 1). It should be noted that $\alpha_m = 0.8$ and $\alpha_c = 1.17$ for the chosen parameters. It is evident that the amplitude of the coarser drops decreases with an increase in α in this case. In the specific instance being examined here, a second coarse drop is formed (curve 1). The decay time t_d for different α is in qualitative agreement with the relation $\sigma(\alpha)$: in the present case, t_d is minimal at $\alpha = 0.8$ ($t_d = 5.6$) and maximal at $\alpha = 1.1$ ($t_d = 6.95$). We took the moment of time when $\min r_*(z) = 0.1$ as the value of t_d.

The calculations showed that a decrease in the initial amplitude δ_c from 0.1 to 0.05 has almost no effect on the results, and the relations $r_*(z)$ coincide to within the graphic representation. An increase in the initial amplitude leads to a significant weakening of the effect of α on the size of the drop. This occurs because the decay time t_d decreases with an increase in δ_c, causing nonlinear effects to play a smaller role in the decay process.

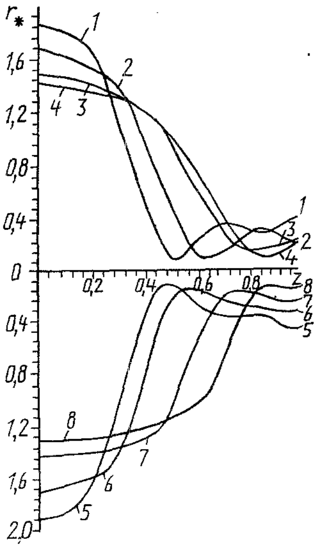

Fig. 1

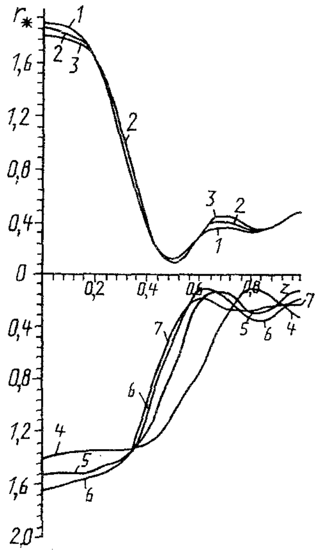

Fig. 2

Fig. 1. Dependence of the form of the surface of a rotating capillary jet $r = r_*(z)$ (with $\omega = 0.36$, $\delta = 0.1$) on q and α: 1-4) with q = 0 and α = 0.6, 0.8, 1.0, and 1.1, respectively; 5-8) with q = 1 and α = 0.6, 0, 8, 1.1, and 1.3, respectively.

Fig. 2. Dependence of the form of a capillary jet $r = r_*(z)$ (with q = 1, $\delta_c = 0.1$) on ω and α: 1-3) $\alpha = 0.6$ and ω = 0; 1.0; 1.96, respectively; 4-7) $\alpha = 1.1$ and ω = 0.64; 1.0; 1.44; 1.96.

Curves 5-8 in Fig. 1 show the relations $r_*(z)$ for q = 1. It should be noted that α_m = 0.96 and α_c = 1.34. The completed calculations showed that an increase in α leads to a decrease in the size of the satellite until its degeneration at $\alpha \to \alpha_c$. An increase in α also leads to a marked decrease in the amplitude of the drop. It should be noted that, compared to the case q = 0, the amplitude of the coarse drop increases with a change in α.

In Fig. 2 (curves 1-3) $\alpha_m(0.64; 1) = 1.04$, while $\alpha_m(1.96; 1) = 1.36$. Despite the increase in α_m, there is no significant change in the results. However, we do see some increase in the amplitude of the satellite.

The effect of rotation on decay increases markedly with an increase in wave number. We will examine the case q = 1, $\delta_c = 0.1$, $\alpha = 1.1$. It is evident from Fig. 2 (curves 4-7) that the dimensions of the satellite increase with an increase in ω and that its structure becomes more complex: one bridge is formed at $\omega = 0.64$, while two bridges are formed at $\omega = 1.0$. There is a simultaneous increase in the amplitude of the coarse drop. The explanation for these results lies in the fact that an increase in ω (for the chosen values of q and α) is accompanied by a transition from the region $\alpha = \alpha_c$ to the region $\alpha \approx \alpha_m$, i.e., the "effective" wave number decreases.

An increase in amplitude δ_c significantly reduces the effect of ω on the decay process at $q \neq 0$ (curves 1-3, Fig. 3). For comparison, Fig. 3 also shows the dashed curve from Fig. 2. As was already noted, an increase in δ_c leads to a decrease in the size of the satellite and its amplitude throughout the wave-number range — especially at $\alpha < \alpha_m$. Thus, the form of the jet before decay becomes less "sensitive" to changes in different parameters in general and to changes in the degree of swirling of the flow ω in particular.

Now let us examine the case when ω becomes constant and the charge q changes with a fixed value of the wave number α. For example, let w = 1, $\alpha = 1$, $\delta_c = 0.1$, while r changes from 0.8 to 1.4. In this case, the value $\alpha = 1$ always remains close to the corresponding value of α_m. As a result, the form of the jet in the final stage of decay turns out to be roughly the same for all numbers within the range 0.8-1.4. Such "conservative" behavior by the jet during its decay was also seen at $\alpha = 1.3$ for the same values of ω and q. However, for perturbations of shorter wavelengths (at $\alpha = 1.5$), an increase in q leads to a substantial increase in the size of the satellite. This can be attributed to the transition of the "effective" wave number from the region near α_c (1.0; 0.8) to the region of maximally growing perturbations α_m (1.0; 1.4).

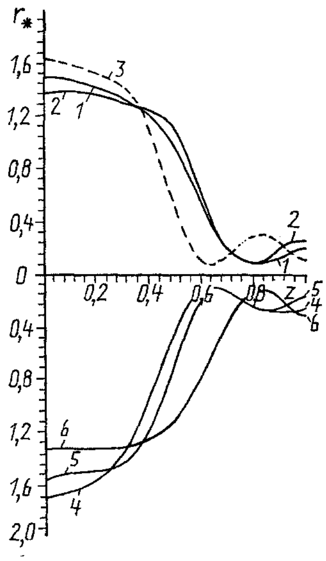

Fig. 3

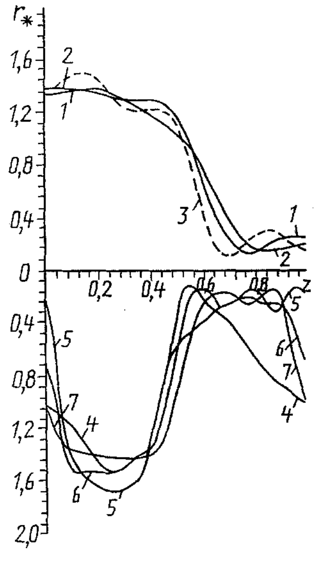

Fig. 4

Fig. 3. Dependence of the form of a capillary jet $r = r_*(z)$ (with $q = 1.0$, $\alpha = 1.1$) on ω for a perturbation of large initial amplitude $\delta_c = 0.2$: 1, 2) $w = 0.64$ and 1.44; with $\delta_0 = 0.1$: 3-7) $\alpha = 1.1$, $\omega = 1.44$; 0.84 and 0; 1.04 and 0.64; 1.14 and 1.0; 1.36 and 1.96.

Fig. 4. Relation for the form of a capillary jet $r = r_*(z)$ (with $\omega = 0.64$; $q = 1.3$, $1 = 0$, $\alpha = 1.2$, $\delta_c = 0.1$) for three values of the time of the sudden change in the charge t_*: 1-3) $t_* = 2.5$; 1.5 and ∞, as well as with a "sawtooth" initial perturbation (at $\omega = 0.64$, $q = 1$; $\delta_c = 0.1$, $\lambda = 5$): 4-7) $\alpha = 0.7$; 0.84; 1.04; 1.20.

At $q = 1$ (curves 4-7, Fig. 3), an increase in ω also leads to a decrease in the size of the satellite. However, in this case the decrease is more substantial (associated with a factor of more than two). The size of the satellite also changes throughout the investigated range of ω; the quantity ω exerts its greatest effect at $\omega = 1.44$. Thus, the range in which ω has an effect expands at $q \neq 0$ and the effect itself intensifies.

To make certain that the results we obtained do not depend on δ_c, we varied this parameter from 0.05 to 0.2. It turned out that with a change in δ_c from 0.1 to 09.05, the results nearly coincide with the data represented by curves 4-7 in Fig. 3. The effect remains with an increase in δ_c form 0.1 to 0.2 but weakens somewhat.

We also examined the possibility of controlling the decay of the swirled jet by varying its charge over time. Here, we will discuss the case of a step function ($q(t < t_*) = q_0$; $q(t > t_*) = 1$). The calculations we performed showed that by "properly" varying t_*, it is possible to also achieve a significant reduction in the size of the satellite up to the point of its degeneration into a kink instability when $\omega \neq 0$.

Figure 4 (curves 1-3) show the results of calculations of the form of the surface of the jet before its disintegration. It should be noted that $t_d = 3.6$ in this case. It is apparent from [4] that a sudden reduction in the charge leads to a decrease in the size of the satellite. Meanwhile, the smaller t_*, the more noticeable the given effect becomes. The value $\alpha = 1.3$ is $\alpha_m(\omega, q_0)$ on the one hand and, on the other hand, nearly coincides with $\alpha_c(\omega, q_1)$. It should be noted that the range of unstable wave numbers for finite-amplitude perturbations becomes broader in this case compared with the linear theory.

The effect of a variable charge decreases with an increase in the initial amplitude δ_c. Thus, with $\delta_c = 0.2$, $\omega = 0.64$, $\alpha = 1.2$, $q_0 = 1.3$, $q_1 = 0$, $t_* = 1.5$, $t_* = 1.0$, the size of the satellite decreases by roughly 10%.

We also examined the case $q_0 < q_1$; let $\omega = 0.64$, $q_0 = 0$, $q_1 = 1.3$. Accordingly, $\alpha_m(0.64; 0) = 0.88$. As with $\omega = 0$, in the given case a sudden increase has the opposite effect — leading to an increase in the size of the satellite.

We also performed calculations for "sawtooth" initial perturbations: $\xi(z, 0) = \xi_2$ (curves 4-7, Fig. 4). It is evident that such initial perturbations lead to a situation whereby the satellite becomes asymmetric relative to its center. This is expressed in the formation of a single constriction instead of two. Thus, during decay of the jet, a small drop may combine with one of the two larger drops adjacent to it. The drop with which the smaller drop will merge depends on the values of the parameters in the specific case. In the example we are examining, with $\alpha = 0.7$ and $\alpha = 1.04$, the satellite joins with the main drop located in front of it (curves 4 and 6). At $\alpha = 0.84$ and $\alpha = 0.2$, no such unions take place (curves 5 and 7). It should also be pointed out that the size of the satellite does not decrease with an increase in α in the case of a "sawtooth" perturbation.

It is also interesting to examine the extent to which the decay process is influenced by the "curvature" of the "saw", i.e., by the parameter λ and the amplitude δ. Calculations were performed with $\omega = 0.64$, $q = 1$, $\alpha = 1.04$, $\delta = 0.1$ for $\lambda = 2$, $\lambda = 5$ and $\lambda = 10$, as an example showed that the differences in the form of the jet before its disintegration were negligible.

Significant differences relative to the initial data for a cosine law were obtained in a study of the effect of the amplitude δ. It turned out that a change in δ may lead to a change not only in the dimensions of the satellite, but also in the location of the constrictions, i.e., the satellite may merge with different adjacent coarse drops. Also, there the dimensions of the satellite do not depend on δ in any certain manner. Finally, the satellite may grow with an increase in δ. It should be remembered that, for cosine perturbations, an increase in δ_c always leads to a decrease in the dimensions of the satellite for all α.

There are certain distinctive features to the decay of a twisted jet in the case of sawtooth perturbations. In particular, compared to the case of cosine initial data, ω has a greater effect on the form and dimensions of the satellite. For example, a substantial increase in the dimensions of the satellite is seen with an increase in ω. As in the case of cosine initial data, this can be attributed to the transition of the wave number from the shortwave region (α close to α_c) to the longwave region ($\alpha \approx \alpha_m$).

Let us also examine the decay of the jet when it develops freely, i.e., when $\alpha = \alpha_m(\omega, q)$.

On the whole, the calculations show that sawtooth initial data affects mainly the formation of the satellite. The changes in the form and dimensions of the main drop are not as substantial compared to the case of cosine initial perturbations. The main feature is the formation of just one constriction on the jet. As a result, the satellite may merge with one of two adjacent drops.

We also studied the effect of the longwave components of the spectrum on the decay process. The need for such study follows from the fact that the perturbation spectrum in actual experiments is continuous.

The effect of the longwave "background" was modeled by assigning an initial perturbation $\xi_0(z)$ in the following form: $\xi_0(z) = \delta b \cos \alpha_b z + \delta_m \cos \alpha_m z$, $\alpha_m = n\alpha_b$, where n is a specified integer. Calculations were performed with the number of basis functions $N = 4n$, $n = 3$.

It is significant that the result obtained with $\omega \neq 0$ and $q \neq 0$ was basically the same as with the case $q = 0$, $\omega = 0$; the effect of the "background" amounts to a decrease in the contribution of the finely dispersed phase. We performed calculations for $\omega = 1.44$, $q = 0.8$; $\alpha_b = 0.39$; $\delta_m = 0.1$; $\delta_b = 0, 0.02$ and 0.05.

The value of ω was varied from 0.36 to 1.96, while q was varied from 0 to 1.2. We also examined the cases $n = 2$ and 4. The maximum number of harmonics here reached $N = 16$. The above effect remained present in all of the variants, while there was a slight change in the structure of the satellites. Also, as previously, we found that the presence of the "background" accelerates the decay of the jet somewhat (the time t_d is shortened by 10-15% with $\delta_b/\delta_m = 0.5$).

Thus, the completed study permits us to conclude that the effect of the longwave "background" in the case of the simultaneous action of a surface charge and twisting of the jet is qualitatively the same as in the case when one of these factors is absent (or both are absent).

Notation. Γ, circulation; θ, polar angle of the cylindrical coordinate system; Φ_c, velocity potential of the main flow; Φ, velocity potential; κ, curvature of the surface; $r = 1 + \xi(z, t)$, equation of the surface of the capillary jet; T, surface tension; α, wave number; α_c, growth factor; Q, surface charge; δ_c, initial amplitude of sinusoidal perturbation of the surface of the capillary jet; α_0, wave number bounding the region of wave-number instability; λ, curvature of the initial sawtooth perturbation; N, number of harmonics; t_d, time of decay; t_*, time during which the surface charge changes abruptly; a, radius of jet; q_1, surface charge per unit length of the jet at the given moment of time; n, external normal to the surface of the jet; α_m, wave number corresponding to the maximum value of α_c.

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EFFECT OF FRICTION ON LOW FREQUENCY SOUND PROPAGATION IN A GAS–LIQUID FOAM

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UDC 532.539+534.19+541.182.45

A model is proposed for propagation of low frequency acoustic disturbances in a gas–liquid foam with consideration of friction on interphase boundaries during liquid motion in a system of interconnected microcapillaries. A Burgers equation with quasilinear convolution-type term is obtained. Structure and dynamics of linear signals are studied over the range of applicability of the model.

The spectrum of technological processes which employ foams and foamlike structures has expanded precipitously and currently encompasses a most varied range of applications [1]. To support production techniques involving foams both in cases where foam formation must be intensified, and in situations where foam disrupts the normal course of a process, a precise realtime knowledge of foam parameters is required. Since one method of solving such problems involves acoustical diagnostics, the problem of determining sound propagation characteristics in foam arises.

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